

Can $X(3872)$ be a $J^P = 2^-$ tetraquark state?

Chun-Yu Cui, Yong-Lu Liu, Guo-Bin Zhang and Ming-Qiu Huang

Department of Physics, National University of Defense Technology, Hunan 410073, China

(Dated: April 30, 2012)

In this article, we test the nature of $X(3872)$, which is assumed to be a P-wave $[cq]$ -scalar-diquark $[\bar{c}\bar{q}]$ -axial-vector-antidiquark tetraquark state with $J^P = 2^-$. The interpolating current representing the $J^P = 2^-$ state is proposed. Technically, contributions of the operators up to dimension six are included in the operator product expansion (OPE). The mass obtained for such state is $m_{2^-} = (4.38 \pm 0.15)$ GeV. We conclude that it is impossible to describe the $X(3872)$ structure as $J^P = 2^-$ tetraquark state.

PACS numbers: 11.55.Hx, 12.38.Lg, 12.39.Mk

The state $X(3872)$ was first discovered by Belle [1] in the $\pi^+\pi^-J/\psi$ mode and then confirmed by the CDF [2], DØ [3], and BABAR [4] Collaborations in the same decay channel. The most recent measure of its mass is [5]

$$m_X = (3871.85 \pm 0.27(\text{stat}) \pm 0.19(\text{syst})) \text{ MeV}, \quad (1)$$

with a width of $\Gamma_X < 1.2$ MeV. Belle [1] and CDF [6] propose that it proceeds through the $X \rightarrow J/\psi\rho \rightarrow J/\psi\pi^+\pi^-$ decay. Since a charmonium state has isospin zero, it can not decay into $X \rightarrow J/\psi\rho$, so the $X(3872)$ is identified as an “exotic” state. According to the CDF analysis of the decay angular distribution [6] and the invariant $\pi^+\pi^-$ mass distribution [7] of the $J/\psi\pi^+\pi^-$ decay mode, only 1^+ and 2^- assignments are possible. The close proximity of $X(3872)$ mass to the $D\bar{D}^*$ threshold indicates that $X(3872)$ might be a loosely bound $D\bar{D}^*$ molecular state, whose quantum number is $J^P = 1^+$. Also, an angular analysis applied to the 2π mass distribution in $J/\psi\rho$ favors the quantum number $J^P = 1^+$ [8]. In compliance with these quantum numbers, many literatures have appeared in the past years. Its possible interpretations include the molecular state, tetraquark state and hybrid charmonium (see reviews [9]-[15] and references therein). Using QCD sum rules (QCDSR) [16], Nielsen *et al.* discuss the possibility that it is possible to describe the $X(3872)$ structure as a mixed molecule-charmonium state and study its strong decay and radiative decay [17, 18].

Very recently, the BABAR collaboration has performed angular distribution analysis of the decay $B \rightarrow J/\psi\omega K$, indicating that P-wave between J/ψ and ω is favored, so that quantum numbers $J^P = 2^-$ is preferred [19]. In this case, the most conventional explanation is the 1^1D_2 charmonium state $\eta_{c2}(1D)$. In Ref [20], the radiative transition processes $\eta_{c2}(1D) \rightarrow J/\psi(\psi') + \gamma$ is investigated within several phenomenological potential models with the assumption that $X(3872)$ is a $\eta_{c2}(1D)$ charmonium, which are in contradiction

with the existing BABAR measurements [21]. The data on its $D^0 \bar{D}^0 \pi^0$ decay mode [22] also contradict the 1^1D_2 charmonium interpretation of the $X(3872)$ [23]. The decay of $B \rightarrow \eta_{c2} X$ is studied in NRQCD factorization framework, which indicates that $X(3872)$ is unlikely to be a 1^1D_2 charmonium state [24]. Thus, we have to resort to exotic explanations for the $J^P = 2^-$ quantum numbers. In Ref. [25], it's shown that the molecular interpretation appears to be untenable, but the tetraquark interpretation may be a viable candidates to be $X(3872)$ with $J^P = 2^-$. Follow their opinion, we study the mass of $X(3872)$ as a P-wave $[cq]$ -scalar-diquark $[\bar{c}\bar{q}]$ -axial-vector-antidiquark tetraquark state with $J^P = 2^-$ using the QCDSR.

The interpolating current representing a $J^P = 2^-$ P-wave tetraquark state with $[cq]$ -scalar-diquark and $[\bar{c}\bar{q}]$ -axial-vector-antidiquark fields is adopted as

$$j_{\mu\nu} = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} [(q_a^T C \gamma_5 c_b) D_\mu (\bar{q}_d \gamma_\nu C \bar{c}_e^T)] . \quad (2)$$

Herein the index T represents matrix transposition, C means the charge conjugation matrix, D^μ denotes the covariant derivative, while a, b, c, d , and e are color indices.

In the QCDSR approach, the mass of the particle can be determined by considering the two-point correlation function

$$\Pi_{\mu\nu,\alpha\beta}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j_{\mu\nu}(x) j_{\alpha\beta}^+(0)] | 0 \rangle . \quad (3)$$

The QCDSR attempts to link the hadron phenomenology with the interactions of quarks and gluons, which is obtained by evaluating the correlation function in two ways: an approximate description of the correlation function in terms of intermediate states through the dispersion relation, a description of the same correlation function in terms of QCD degrees of freedom via OPE.

In the phenomenological side, the correlation function is calculated by inserting a complete set of intermediate states with the same quantum numbers as the tetraquark state. Parametrizing the coupling of the $J^P = 2^-$ tensor state to the current $j_{\mu\nu}$ in term of the parameter f_X as

$$\langle 0 | j_{\mu\nu}(0) | X \rangle = f_X \varepsilon_{\mu\nu}, \quad (4)$$

where $\varepsilon_{\mu\nu}$ is the relevant polarization tensor. Using Eq. (4) in the phenomenological side of Eq. (3), we obtain

$$\Pi_{\mu\nu,\alpha\beta} = \frac{f_X^2}{m_X^2 - q^2} \left\{ \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \right\} + \text{other structures} + \dots, \quad (5)$$

where the only structure which contains the contribution of the tensor meson has been kept. In calculations, we have performed summation over the polarization tensor using

$$\varepsilon_{\mu\nu} \varepsilon_{\alpha\beta}^* = \frac{1}{2} T_{\mu\alpha} T_{\nu\beta} + \frac{1}{2} T_{\mu\beta} T_{\nu\alpha} - \frac{1}{3} T_{\mu\nu} T_{\alpha\beta}, \quad (6)$$

with

$$T_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_X^2}. \quad (7)$$

In the OPE side, we single out the structure $\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})$ whose coefficient is denoted as

$$\Pi^{(1)}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2}, \quad (8)$$

where the spectral density is $\rho^{OPE}(s) = \frac{1}{\pi} \text{Im} \Pi^{(1)}(s)$. After equating the two sides, assuming quark-hadron duality, and making a Borel transformation, the sum rule can be written as

$$f_X^2 e^{-m_X^2/M^2} = \int_{4m_c^2}^{s_0} ds \rho^{OPE}(s) e^{-s/M^2}, \quad (9)$$

with M^2 the Borel parameter.

In calculations, we work at the leading order in α_s and consider vacuum condensates up to dimension six, with the similar techniques in Refs. [26, 27]. After tedious calculation, the concrete forms of spectral densities read:

$$\rho^{OPE}(s) = \rho^{\text{pert}}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{\langle g^2 G^2 \rangle}(s) + \rho^{\langle g\bar{q}\sigma \cdot Gq \rangle}(s) + \rho^{\langle \bar{q}q \rangle^2}(s), \quad (10)$$

with

$$\begin{aligned} \rho^{\text{pert}}(s) &= \frac{1}{5 * 2^{13} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (\alpha^3 + \alpha^2\beta - 2\alpha^2 + \alpha\beta^2 - 3\alpha\beta + 2\alpha + \beta^3 \\ &\quad - \beta^2 + \beta - 1) r(m_c, s)^5, \\ \rho^{\langle \bar{q}q \rangle}(s) &= \frac{\langle \bar{q}q \rangle}{3 * 2^8 \pi^4} m_c \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\beta^2 + 2\alpha\beta - 2\beta - 1) r(m_c, s)^3, \\ \rho^{\langle g^2 G^2 \rangle}(s) &= \frac{\langle g^2 G^2 \rangle}{3^3 * 2^{13} \pi^6} m_c^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} (\alpha + \beta - 1)^2 (2\alpha^2 + \alpha - 2\beta^2 - \beta) r(m_c, s)^2, \\ \rho^{\langle g\bar{q}\sigma \cdot Gq \rangle}(s) &= \frac{\langle g\bar{q}\sigma \cdot Gq \rangle}{2^{10} \pi^4} m_c \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha(\alpha - 1)} [m_c^2 - \alpha(1 - \alpha)s]^2 \\ &\quad - \frac{\langle g\bar{q}\sigma \cdot Gq \rangle}{2^{10} \pi^4} m_c \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (4\alpha^2\beta + 6\alpha\beta^2 - 2\alpha\beta - \alpha^2 - \beta) r(m_c, s)^2, \\ &\quad - \frac{\langle g\bar{q}\sigma \cdot Gq \rangle}{2^{10} \pi^4} m_c \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\beta^2 + 2\alpha\beta - 2\beta - \alpha) r(m_c, s)^2, \\ \rho^{\langle \bar{q}q \rangle^2}(s) &= \frac{\langle \bar{q}q \rangle^2}{3 * 2^5 \pi^2} m_c^2 \left(\frac{2m_c^2}{3} - \frac{s}{6} \right) \sqrt{1 - 4m_c^2/s}, \end{aligned} \quad (11)$$

with $r(m_c, s) = (\alpha + \beta)m_c^2 - \alpha\beta s$. The integration limits are given by $\alpha_{\min} = \left(1 - \sqrt{1 - 4m_c^2/s}\right)/2$, $\alpha_{\max} = \left(1 + \sqrt{1 - 4m_c^2/s}\right)/2$, and $\beta_{\min} = \alpha m_c^2/(s\alpha - m_c^2)$.

TABLE I: Upper limits in the Borel window obtained from the sum rule for different values of $\sqrt{s_0}$.

$\sqrt{s_0}$ (GeV)	M_{max}^2 (GeV ²)
4.6	2.3
4.7	2.4
4.8	2.6
4.9	2.8
5.0	2.9

To extract the mass m_X , we take the derivative of Eq.(9) with respect to $\frac{1}{M^2}$ and then divide the result by itself

$$m_X^2 = \int_{4m_c^2}^{s_0} ds \rho^{OPE}(s) s e^{-s/M^2} / \int_{4m_c^2}^{s_0} ds \rho^{OPE}(s) e^{-s/M^2}. \quad (12)$$

Before the numerical analysis of Eq.(12), we first specify the input parameters. The quark mass is taken as $m_c = 1.23$ GeV [28]. The condensates are $\langle \bar{q}q \rangle = -(0.23)^3$ GeV³, $\langle g \bar{q} \sigma \cdot G q \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = 0.8$ GeV², and $\langle g^2 G^2 \rangle = 0.88$ GeV⁴ [16]. Complying with the standard procedure of the QCDSR, the threshold s_0 and Borel parameter M^2 are varied to find the optimal stability window. There are two criteria (pole dominance and convergence of the OPE) for choosing the Borel parameter M^2 and threshold s_0 . In general, the continuum threshold s_0 is a parameter of the calculation which is connected to the mass of the studied state, by the relation $\sqrt{s_0} \approx (m_X + 0.5 \text{ GeV})$.

Concretely, the contributions from the high dimension vacuum condensates in the OPE are shown in Fig.1. We have used $\sqrt{s_0} \geq 4.6$ GeV. From this figure it can be seen that for $M^2 \geq 2.0$ GeV², the contribution of the dimension-6 condensate is less than 16% of the total contribution and the contribution of the dimension-5 condensate is less than 20% of the total contribution, which indicate a good Borel convergence. Therefore, we fix the uniform lower value of M^2 in the sum rule window as $M_{min}^2 = 2.0$ GeV². The upper limit of M^2 is determined by imposing that the pole contribution should be larger than continuum contribution. Fig. 2 demonstrates the contributions from the pole terms with variation of the Borel parameter M^2 . We show in Table I the values of M_{max}^2 for several values of $\sqrt{s_0}$. In Fig.3, we plot the tetraquark state mass in the relevant sum rule window, for different values of $\sqrt{s_0}$. It can be seen that the mass is very stable in the Borel window with the corresponding threshold $\sqrt{s_0}$. The final estimate of the $J^P = 2^-$ tetraquark state is obtained as

$$m_X = (4.38 \pm 0.15) \text{ GeV}. \quad (13)$$

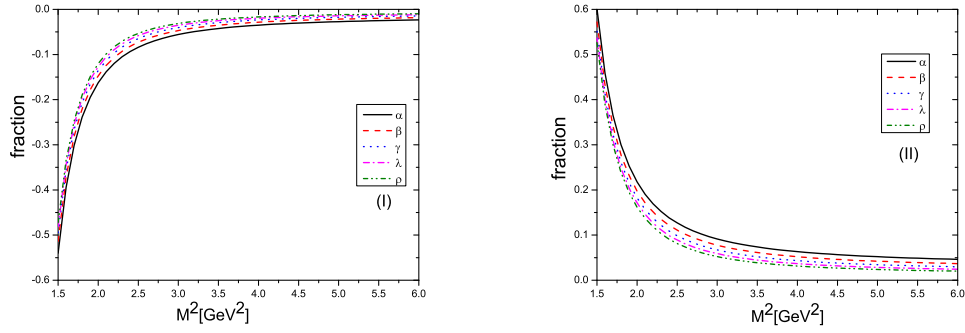


FIG. 1: The OPE convergence for the $J^P = 2^-$ tetraquark state. The I and II correspond to the contributions from the $D = 6$ term and the $D = 5$ term, respectively. Notations α , β , γ , λ and ρ correspond to threshold parameters $\sqrt{s_0} = 4.6 \text{ GeV}$, 4.7 GeV , 4.8 GeV , 4.9 GeV and 5.0 GeV , respectively.

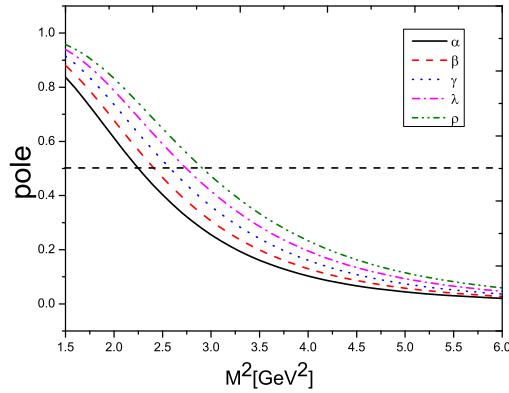


FIG. 2: Contributions from pole terms with variation of the Borel parameter M^2 in the case of $J^P = 2^-$ tetraquark state. Notations α , β , γ , λ and ρ correspond to threshold parameters $\sqrt{s_0} = 4.6 \text{ GeV}$, 4.7 GeV , 4.8 GeV , 4.9 GeV and 5.0 GeV , respectively.

In summary, by assuming $X(3872)$ as a $[cq][\bar{c}\bar{q}]$ tetraquark state with quantum numbers $J^P = 2^-$, the QCDSR approach has been applied to calculate the mass of the resonance. Our numerical results are $m_X = (4.38 \pm 0.15) \text{ GeV}$, which indicates that $X(3872)$ is unlikely to be a $J^P = 2^-$ tetraquark state. Thus, $J^P = 1^+$ assignment for the quantum numbers of the $X(3872)$ is favored.

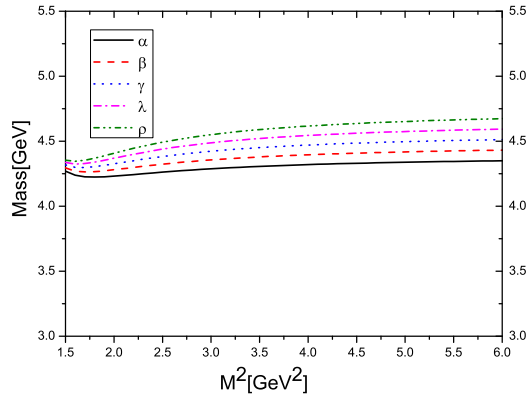


FIG. 3: The mass of the $J^P = 2^-$ tetraquark state as a function of M^2 . Notations α , β , γ , λ and ρ correspond to threshold parameters $\sqrt{s_0} = 4.6$ GeV, 4.7 GeV, 4.8 GeV, 4.9 GeV and 5.0 GeV, respectively.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Contracts Nos.10975184, 11047117, 11105222 and 11105223.

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